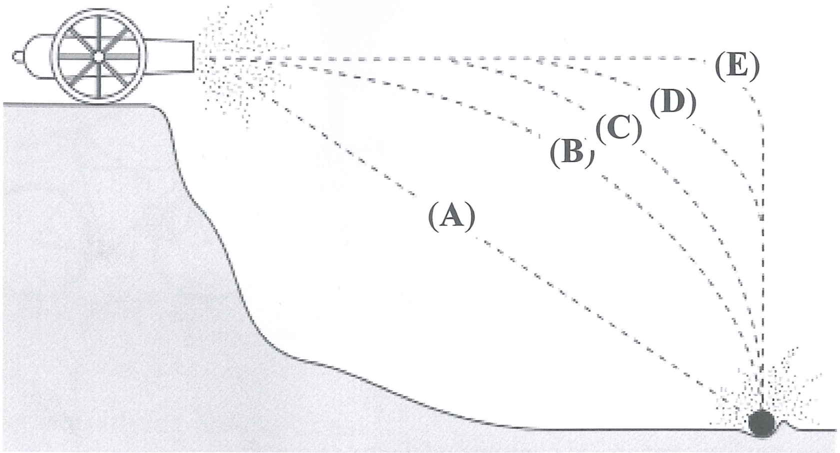
Exam
Mechanics & Relativity 2013–2014 (part Classical Mechanics)
Januari 23, 2014

Problem 1 Below are four conceptual multiple-choice questions. Only the answer matters, not your arguments.

a. A ball is fired by a cannon from the top of a cliff as shown in the figure below. Which of the paths would the cannon ball most closely follow?



b. A boy throws a steel ball straight up. Consider the motion of the ball only after it has left the boy's hand but before it touches the ground, and assume that forces exerted by the air are negligible. For these conditions, the force(s) acting on the ball is (are):

A) a downward force of gravity along with a steadily decreasing upward force.

B) a steadily decreasing upward force from the moment it leaves the boys hand until it reaches its highest point; on the way down there is a steadily increasing downward force of gravity as the object gets closer to the earth.

C) an almost constant downward force of gravity along with an upward force that steadily decreases until the ball reaches its highest point; on the way down there is only a constant downward force of gravity.

D) an almost constant downward force of gravity only.

E) none of the above. The ball falls back to ground because of its natural tendency to rest on the surface of the earth.

c. The figure below shows a boy swinging on a rope, starting at a point higher than P. Consider the following distinct forces:

1. a downward force of gravity.

2. a force exerted by the rope pointing from P to O.

3. a force in the direction of the boys motion.

4. a force pointing from O to P.

Which of the above forces is (are) acting *on* the boy when he is at position P?

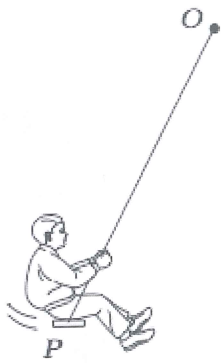
A) 1 only.

B) 1 and 2.

C) 1 and 3.

D) 1,2 and 3.

E) 1, 3 and 4.



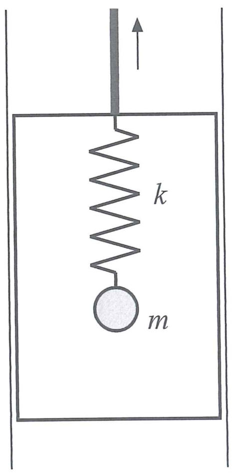
d.

A mass–spring system is suspended from the ceiling of an elevator. When the elevator accelerates upwards, the frequency of this harmonic oscillator,

A) decreases

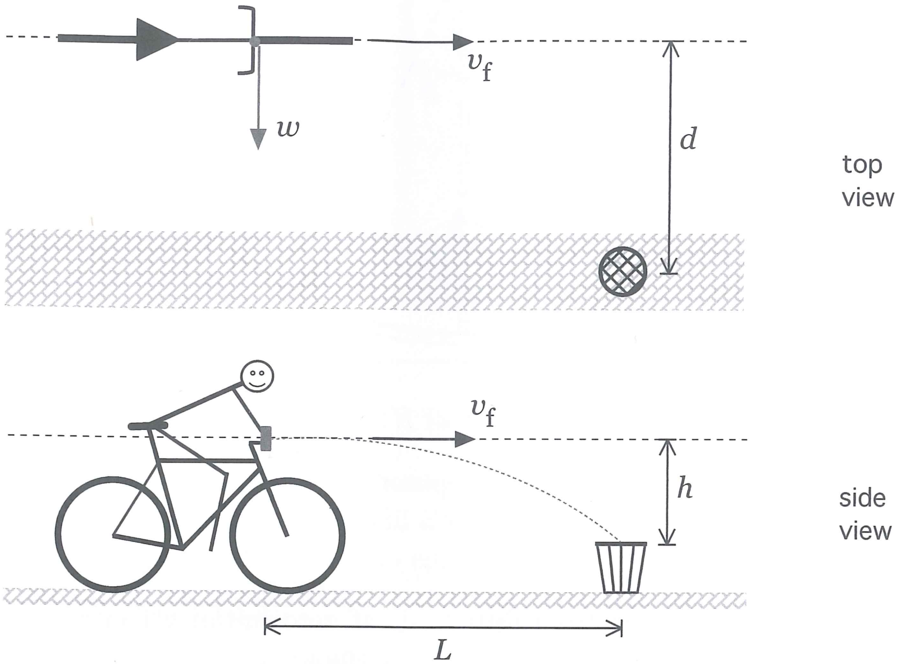
B) increases

C) remains the same



The answers to problems 2 through 4 require clear arguments and derivations, all written in a well-readable manner.

Problem 2 You are riding your bicycle at a constant speed v_f, holding an empty beverage can that you want to get rid of. By the side of the road ahead, you notice a bin and you decide to toss the can by throwing it to the side. When should you do so, and with what velocity in order that the can falls into the bin?



With the following simplifying assumptions:

- you keep riding parallel to the curb, and the distance between the bin and your route is d;
- you throw the can in the horizontal direction with a velocity w perpendicular to your route;
- the dimensions of the can and the bin can be neglected;
- there is no air resistance;
- the can is launched from the mid-plane of the bike;
- the height difference between the initial position of the can and the top of the bin is h,

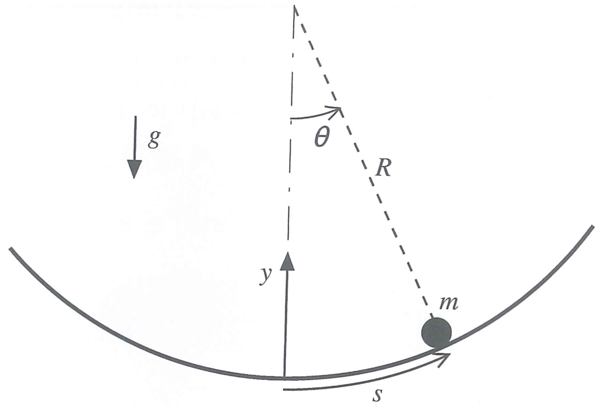
we can now analyse this problem.

a. Suppose we would already know the distance L at which you should toss the can, with which sideways velocity w should you do so?

b. Given the initial velocity components v_f and w in the horizontal plane, how much time does it take for the can to fall *down* into the bin?

c. At what distance L should you launch the can with speed w so that it ends up in the bin?

Problem 3 A ball with mass m can slide without friction inside a ring of radius R. The position of the ball is measured by the arc length $s = R\theta$, with θ the angle relative to the vertical axis.



a. Determine the kinetic energy of the ball $K(\dot{\theta})$ [Note the dimensions!].

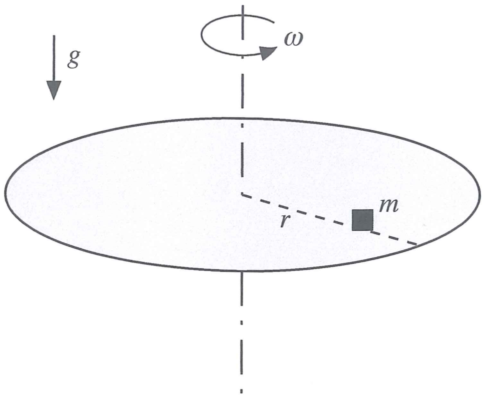
b. Show that, for small angles θ (‡), the potential energy of the ball due to gravity g can be written in the same form as for a linear spring, namely $V(\theta) = \frac{1}{2}k\theta^2$. Express the "spring constant" k in terms of the quantities given above.

c. When the ball is released from $\theta(0) \equiv \theta_0$ at zero initial velocity, it will oscillate about the equilibrium position $s = 0$ in a harmonic fashion. Derive the equation of motion by making use of the fact that the energy $E = K + V$ remains constant at all times.
Show that the angular frequency of this harmonic motion is given by $\omega = \sqrt{g/R}$.

d. Determine the velocity of the ball at the instant it passes by the equilibrium position $\theta = 0$.
Compare this answer with the velocity of the ball at the same height $y = 0$ (see figure) in case it had been dropped from the same height $y = R(1 - \cos\theta_0)$ outside the ring. Explain your answer.

‡In that case: $\sin\theta \approx \theta$ and $\cos\theta \approx 1 - \frac{1}{2}\theta^2$.

Problem 4 A small beetle of mass m is standing on a disc that rotates about a vertical axis at constant angular frequency ω. The coefficient of friction between beetle and disc is μ.



a. Which forces does the beetle observe (in his co-moving frame of reference) when it is positioned at a distance r from the axis? Specify direction and magnitude of all forces. Determine the maximum velocity of the disc for which the beetle is able to remain at rest on the disc.

b. Describe the forces when he is not at rest but moves radially towards the center of the disc. Draw these forces in a free-body diagram of the beetle.

Problem 5 Bob and Ann disagree about the sign in the expression for the potential energy of a spring with linear stiffness k. Bob argues as follows:

- In order to stretch the spring over a distance x, I have to apply a force of magnitude kx in the direction of x;
- Substitution of this in the definition of potential energy $V(x)$ then yields

$$\begin{aligned} V(x) &= -\int_0^x F(x')dx' \\ &= -k\int_0^x x'dx' = \\ &= -\frac{1}{2}kx^2 \,. \end{aligned}$$

However, according to Ann, the potential energy of a spring is given by $V = +\frac{1}{2}kx^2$.
Question: who is right, and why?

Grading:

Question	# of points
1	4
2	3
3	6
4	2
5	1

Exam grade = (total # points + 1) /1.7